An improved approach for thaw depth evaluation considering unfrozen water in frozen soil

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Abstract

The thaw depth of permafrost is known to depend on the ice content in frozen soil. Current engineering practice of thaw depth calculation often ignores the unfrozen water in permafrost and assumes that all water in the soil completely freezes below the water freezing temperature (e.g., 0 °C). Fine-grained soil typically contains silts and clays and may have significant amounts of unfrozen water below freezing temperature. The assumption of ignoring the unfrozen water leads to a less conservative estimation of the thaw depth by typically about 10% to 20% (up to 30%) for fine-grained soils. There is a need to develop a practical and industry-friendly approach for thaw depth assessment, which takes account of the unfrozen water content in frozen soil. The studies by Tice et al. (1976), Anderson and Ladanyi (2003) and Qu and Pham (2023) suggested a correlation between unfrozen water content in frozen soil and temperature below freezing point. The correlation (Qu and Pham 2023) can be established using the liquid limit and total water content of soil, which are available for most commercial projects. This paper presents an improved approach for thaw depth evaluation to take account of the unfrozen water content in frozen soil using the correlation associated with temperature.

1 Introduction

The current engineering practice to calculate thaw depth often ignores the unfrozen water and assumes that all water in the soil completely freezes below 0 °C (see Canadian Geotechnical Society 2006 and Bianchini and Gonzalez 2012). The volumetric latent heat of fusion, as a key parameter for the thaw depth (see the modified Berggren equation in Canadian Geotechnical Society 2006) is governed by the content of the ice in soil, which can be thawed into water. This assumption leads to underestimating the thaw depth, particularly for fine-grained soils, where the water in soil freezes over a wide range of temperatures from -20 °C to 0 °C. Underestimation of thaw depth could lead to unsafe design for piles and potential thaw settlement for infrastructures in permafrost areas.

This paper presents an improved and practical approach for thaw depth evaluation by taking into account the impact of unfrozen water in frozen soil. This approach adopted the correlation of unfrozen water content with temperature based on the studies by Tice et al. (1976), Anderson and Ladanyi (2003) and Qu and Pham (2023). This approach intends to support a preliminary assessment of thaw depth in engineering practice and requires only inputs from the routine standard laboratory test data. For critical projects, design engineers may consider a field test (thermistor) or advanced laboratory tests, for example, nuclear magnetic resonance (NMR) test.

2 Background

2.1 General

The theoretical methodology for thaw depth has been well established and the engineering practice usually adopts the modified Berggren equation, as recommended in the Canadian Foundation Engineering Manual (CFEM; Canadian Geotechnical Society 2006) and Bianchini and Gonzalez (2012). Equation [1] shows the formula.

$$X = \lambda\sqrt{2k_s I_s / L_s} \qquad [1]$$

where

X = thaw depth
I_s = surface thaw index
k_s= thermal conductivity of the soil
L_s= volumetric latent heat of the soil
λ = a dimensionless coefficient

Modified Berggren equation was developed using a series of assumptions and simplifications. The key simplifications are described below:

1. The mathematical model assumes one dimensional heat flow with the soil mass.
2. In the initial state prior to thaw, the soil mass has a uniform temperature of T_g, i.e., the mean annual ground temperature. Figure 1 shows the initial state.
3. In the thawing state, the top surface of the soil mass has a temperature of T_t, i.e., the mean temperature during the thaw season. Figure 3 shows the thaw season duration (see the shaded area) state.
4. The surface temperature of the soil mass changes suddenly as a step function from T_g to T_t and remains at T_t throughout the period (t_t in days) of the thawing season. Figure 2 shows the simplified temperature change.

Figure 3 illustrates the ground temperature change, and the physical meaning of T_g, T_t, and t_t.

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There are two key factors of soil mass impacting the thaw depth. One is the latent heat required to thaw the ice in frozen soil mass. The other one is the volumetric heat of the frozen soil.

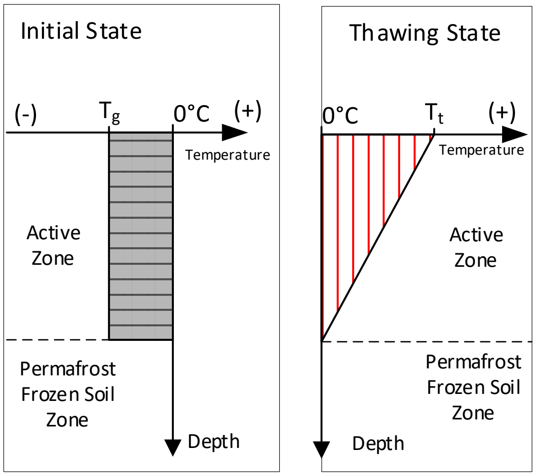

Figure 1. Initial and thawing state in mathematical model for thaw depth calculation.

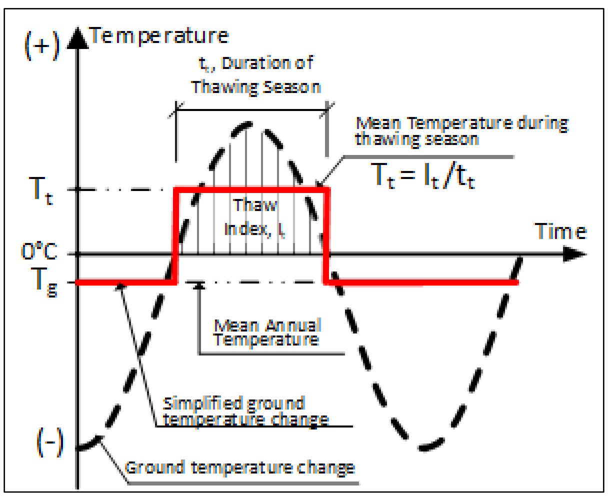

Figure 2. Simplified temperature change between initial state and thawing state (modified Andersland and Ladanyi 2003).

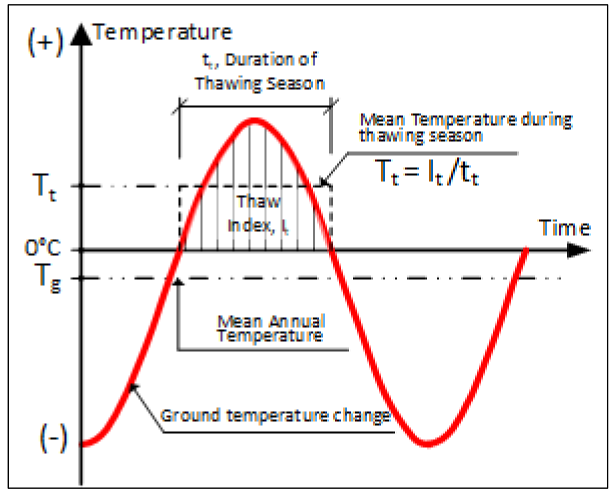

Figure 3. Typical Ground Temperature Change and Physical Meaning of T_g, T_t, and t_t (modified from Andersland and Ladanyi 2003).

One key input for the calculation is the volumetric latent heat of fusion (L_s) for soil, which is the amount of energy required to thaw the frozen water in soil to the thawed state. Therefore, the actual frozen water content in frozen soil has a significant impact to the latent heat and consequently the thaw depth. L_s can be obtained from the formula below:

$$L_s = \gamma_d w_i L \quad [2a]$$

where

L_s = volumetric latent heat of fusion

γ_d = dry unit weight of soil

w_i= frozen water (i.e., ice) content in frozen soil. $w_i = w_c$ if ignoring the unfrozen water in frozen soil, where w_c is total water content (including both frozen and unfrozen water) in soil.

L= latent heat of fusion between water and ice, 334 kJ/kg

The ice content in frozen soil within the thaw depth can be estimated using the equation below:

$$w_i = \; w_c - w_u(T_g) \quad [2b]$$

where T_g represents the mean annual ground temperature in °C. It is less than $0°\text{C}$ in permafrost areas. w_c is total water content (including both frozen and unfrozen water) in soil. $w_u(T_g)$ is the unfrozen water content and can be calculated using Eq. [3].

The calculation approach has been widely adopted in industry, as recommended by Canadian Geotechnical Society (2006) and Bianchini and Gonzalez (2012). As highlighted in the CFEM (Canadian Geotechnical Society 2006), the assumption of ignoring the unfrozen water in frozen soil ($w_i = w_c$) will lead to the unconservative design for both thaw depth and thaw depth evaluation.

3 Unfrozen Water Content In Frozen Soil

The studies have been carried out to quantify the unfrozen water content in frozen soil using correlation with specific area of soil (Anderson and Tice 1972; Anderson and Morgenstern 1973; Anderson et al. 1973) or liquid limit index (Tice et al. 1976; Anderson and Ladanyi 2003; Qu and Pham 2023). The specific surface area refers to the total surface area contained in a unit mass of soil. The correlation approach based on specific area was not widely adopted as the lab test to determine specific area of soil is often not available for most engineering projects.

Tice et al. (1976) first proposed the correlation with liquid limit and obtained a good match with the lab tests. This approach requires the input parameters of $LL_{N=25}$ (Liquid Limit) and $LL_{N=100}$ from non-standard lab tests (i.e., non-standard liquid limit test of N=100, N is the number of blows required to close the standard groove in the liquid-limit test). The requirement of $LL_{N=100}$ potentially limited the application of this approach in industry.

Qu and Pham (2023) proposed an improved approach based on the framework by Tice et al. (1976). For the improved approach, the input parameters can be obtained based on the routine laboratory tests (liquid limit and water content). The study by Qu and Pham (2023) shows that a reasonable agreement was obtained for the improved correlation and the laboratory measurement data of the unfrozen water content in frozen soil.

The following presents the correlation proposed by Qu and Pham (2023) to estimate the unfrozen water content in frozen soil.

$$w_u(T) = \begin{cases} w_c - (w_c - w_{u1}) \times (-T) & for\ T \in (0, -1℃) \quad [3a] \\ w_{u1} \times (-T)^{\beta} & for\ T < -1℃ \quad [3b] \end{cases}$$

[3]

$$w_{u1} = w_{u,T=-1℃} = 0.35 \times LL - 3\% \quad [4]$$

where

T is the temperature in Celsius degree (℃).

w_{u1} is the unfrozen water content in frozen soil at $T = -1℃$, which can be estimated using the equation [4] for preliminary evaluation or specific tests.

β is a material parameter with a default value of -0.25 Qu and Pham (2023) provide further discussion of sensitivity studies for this parameter.

Figure 4 illustrates the correlation from Eq. [3]. In general, the unfrozen water content (w_u) decreases with lower temperature. For temperature from 0 to -1 °C, w_u reduces linearly from the total water content, w_c to w_{u1}, see

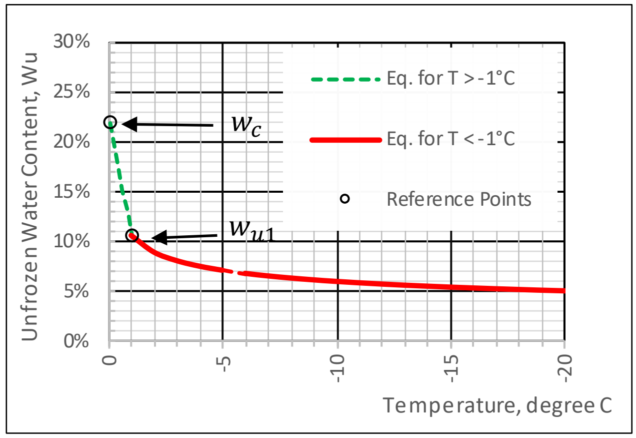

Figure 4. Illustration of correlation (Equation [3]) between unfrozen water content in frozen soil and temperature. (Notes:w_c=22%, LL=40%, α= w_{u1}=10.6%, β= -0.25).

Eq. [3a]. Below -1 °C, the w_u reduction is in a linear log-log correlation which is governed by the slope of the log-log plot, β, see Eq. [3b].

The following describes the main features of the improved approach by Qu and Pham (2023).

1. The improved approach eliminates the requirement for non-standard liquid limit tests, see Eq. [3b]. The parameters for this approach can be obtained based on the routine laboratory tests (liquid limit and water content).
2. For the temperature range from 0 to $-1℃$, the proposed approach adopts a linear correlation of unfrozen water content from the total water content (w_c) to w_{u1}, as shown in Eq. [3a] and Figure 4. For the formula proposed by Tice et al. (1976), the w_u approaches to an infinitely high value as the temperature approaches 0℃, leading to potential numerical issues in analysis.

4 Thaw Depth Calculation

This section presents the proposed procedure in the thaw depth calculation (i.e., modified Berggren approach) to consider the unfrozen soil water content.

Step 1 – Prepare site-specific climate data.

This step is to obtain the key climate parameters, T_g, T_t, and t_t (see Section 2 for details).

Step 2 - Prepare soil data.

This step is to characterize the soil at site including soil type, liquid limit, dry unit weight, and total water content (w_c).

Step 3 – Evaluate surface n-factor.

The step is to assess the site-specific ground condition and evaluate the surface n-factors. The assessment of n-factors is beyond the scope of this paper. Detailed evaluation of N-factors is summarized by Anderson and Ladanyi (2003) and Canadian Geotechnical Society (2006).

Step 4 – Estimate ice content in the initial state and other parameters.

The step is to calculate the ice content in the frozen soil at the initial state corresponding to T_g using Eq. [3] and Eq. [4]. The other parameters such as soil thermal conductivity and latent heart can be obtained using the formula recommended by Canadian Geotechnical Society (2006) or Bianchini and Gonzalez (2012).

Step 5 – Evaluate thaw depth in the thawing state.

This step is to calculate the thaw depth using the modified Berggren approach with the inputs from steps 1 to 4.

Canadian Geotechnical Society (2006) provides a series of graphs to assist engineers for manual calculation using the modified Berggren approach. Qu (2023) provides a step-by-step calculation template in both MatchCAD and Excel formats, which are available for download at the website listed in the reference.

5 APPLICATION AND DISCUSSIONS

This section shows the application of the proposed approach of thaw depth calculation and illustrate the impact of the unfrozen water factor to the calculated thaw depth.

Two representative sites were selected for the thaw depth calculation. One site is located at cold permafrost with the mean annual ground temperature (MAGT) of -14.6 °C and the other site is located at warm permafrost with MAGT of -2 °C. Table 1 lists the parameters for these two sites.

Table 1. Summary of site conditions in sensitivity study. Note: The soils at Site 1 and Site 2 are fine-grained soil.

Parameters	Site 1	Site 2
I_a (ºC -days)	1984	425
nt	1.2	1.2
w_c (%)	22%	22%
LL(%)	40%	40%
γ_s (kN/m^3)	17	17
MAGT (ºC)	-4.3	-14.6
MAGT (ºC)	-2	-9.5
t (days)	160	102

Table 2 presents the calculated thaw depths. Figure 5 shows the results. The following summarizes the findings.

1. Site 1 has a much deeper thaw depth than Site 2 due to its colder MAGT.
2. The assumption of ignoring unfrozen water would lead to an *underestimation* of thaw depth, by an arrange from 21% to 11% for Site 1 and Site 2 respectively.

Table 2. Summary of calculated thaw depths. Note: See the parameters in Table 1.

Cases	Site 1	Site 2
Case accounting for unfrozen water **(base case)**	2.84 m (100%)	1.01 m (100%)
Case Ignoring w_u	2.25 m (79%)	0.90 m (89%)

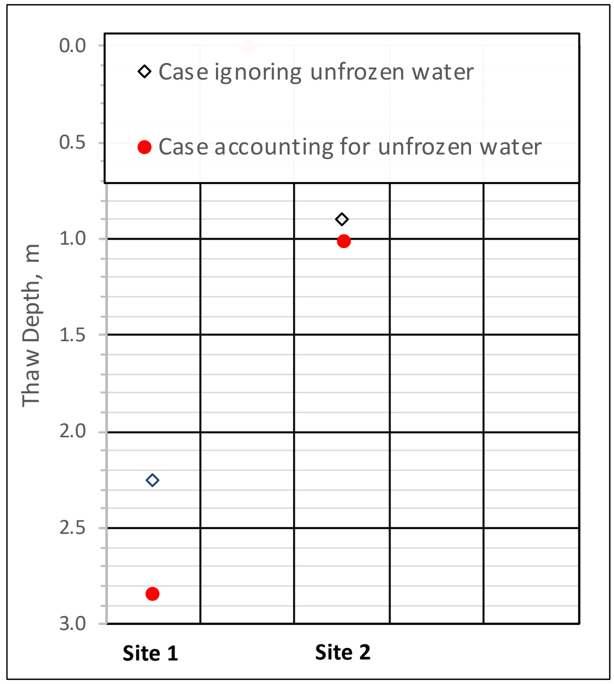

Figure 5. Sensitivity of thaw depth at three sites for four cases: (1) assuming fully frozen water (ignoring Wu), and (2) considering unfrozen water in frozen soil.

Figure 6 shows the calculated thaw depths according to different total water content. The thaw depth ratio was used to evaluate the impact of unfrozen water to the thaw depth. This represents the ratio of the thaw depth ignoring unfrozen water to that accounting for unfrozen water. The following summarizes the findings.

With a wider range of w_c from 15% to 30%, the assumption of ignoring unfrozen water would lead to an underestimation of thaw depth, by an arrange from 31% to 9%.

The impact of ignoring unfrozen water to the thaw depth increases with the higher total water content for both Site 1 and Site 2.

The impact of unfrozen water is more significant for Site 1 at warm permafrost, in comparison with the cold permafrost Site 2.

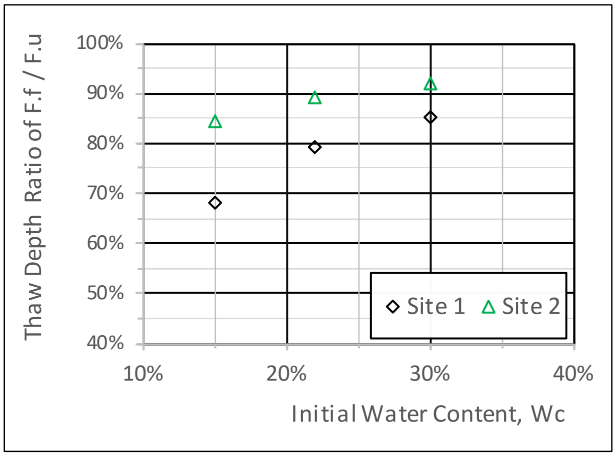

Figure 6. Sensitivity of thaw depth with initial water content (thaw depth ratio: thaw depth for the case ignoring Wu divided by that for the case considering Wu (unfrozen water in frozen soil) using Eq. [3]).

6 Summary and Conclusions

This paper presented an improved approach for engineers to calculate the thaw depth considering the unfrozen water in frozen soil. In comparison with the conventional approach which ignores the unfrozen water in frozen soil below freezing temperature, this approach provides a more conservative thaw depth (by about 10% to 20%) for the fine-grain soil. The thaw depth calculation approach was coded in Mathcad and Excel formats and is available for download at the website listed in the reference (Qu 2023). The following summarizes the main findings.

1. The proposed approach is practical as it requires only two basic input parameters, i.e., total initial water content and liquid limit, which can be obtained from routine standard tests and are usually available in most projects.
2. The proposed approach would lead to a more conservative thaw depth (typically about 10% to 20%, up to 30%) for fine-grained soil, in comparison to the conventional approach ignoring the unfrozen water in frozen soil. For soil with a lower water content, the proposed approach yields a similar thaw depth estimate with the conventional approach as the unfrozen water content is relatively low in this case.
3. If salts are present in soil, a correction should be applied to the proposed approach for thaw depth calculation, depending on the salinity measurement from the site.
4. Future studies using laboratory or field tests will be helpful to verify the findings presented in the paper regarding the impact of unfrozen water in frozen soil on thaw depth.

7 List of Parameters

N: the number of blows required to close the standard groove in the liquid-limit test.

w_c : total water content in frozen soil, including both unfrozen and frozen water.

w_u : unfrozen water content in frozen soil.

w_{u_avg}: average unfrozen water content in frozen soil.

w_i: ice content in frozen soil.

γ_d: dry soil unit weight.

γ_s: bulk soil unit weight.

γ_w: unit weight of water.

I_a: air freezing index

I_s: surface freezing index.

L: latent heat of fusion of water to ice, typical value of 334 kJ/kg.

L_s: volumetric latent heat of the soil.

k_f thermal conductivity of the frozen soil.

w_{u1} : unfrozen water content in frozen soil at $T = -1$℃. It can be estimated using the equation [4].

β: soil parameter for correlation between unfrozen water content in frozen soil and temperature, depending on the soil types, see Eq. [3].

LL: Liquid limit of soil.

T: Temperature in Celsius, degree.

θ: Absolute value of temperature in Celsius, degree (always a positive value).

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